

# Diquarks and Density

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**Abstract.** We describe aspects of the role that diquark correlations play in understanding baryon structure and interactions. The significance of diquarks in that application motivates a study of the possibility that dense hadronic matter may exhibit diquark condensation; i.e., quark-quark pairing promoted by a quark chemical potential. A Gorkov-Nambu-like gap equation is introduced for QCD and analysed for 2-colour QCD (QC<sub>2</sub>D) and, in two qualitatively different truncations, for QCD itself. Among other interesting features, we illustrate that QC<sub>2</sub>D with massive fermions undergoes a second-order transition to a superfluid phase when the chemical potential exceeds  $m_\pi/2$ . In the QCD application we illustrate that the  $\sigma := -\langle \bar{q}q \rangle^{1/3} \neq 0$  phase, which determines the properties of the mass spectrum at zero temperature and chemical potential, is unstable with respect to the superfluid phase when the chemical potential exceeds  $\approx 2\sigma$ , and that at this point the diquark gap is large,  $\approx \sigma/2$ . The superfluid phase survives to temperatures greater than that expected in the core of compact stars.

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## 1 Diquarks

A diquark is a bosonic quark-quark correlation, which is necessarily coloured in all but 2-colour QCD (QC<sub>2</sub>D). Therefore, in the presence of colour-confinement, diquarks cannot be directly observed in a  $N_c \geq 3$  colour gauge theory's spectrum. Nevertheless evidence is accumulating that suggests confined diquark correlations play an important role in hadronic spectroscopy and interactions.

The first discussion of diquark correlations in literature addressing the strong interaction is almost coincident with that of quarks themselves [1,2]. It was quickly realised that both Lorentz scalar and vector diquarks, at least, are important for baryon spectroscopy [3] and, from a consideration of baryon magnetic moments [4], that the diquark correlations are not pointlike. This latter point is still often overlooked, although with decreasing frequency and now certainly without the imputation that it is a realistic simplification.

The motivation for considering diquarks in the constituent-quark model is that treating baryons directly as a three-body problem poses significant challenges in anything other than a mean-field approach. The task is much simplified if two of the constituents can be replaced by a single degree of freedom. However, an obvious question is whether there is any sense in which that replacement is more than just an expedient; i.e., a sense in which it captures some important aspect of QCD's dynamics? The answer is “yes” and we now turn to explaining that.

A significant step toward a description of baryons in quantum field theory can be identified in the realisation [5] that a large class of field theoretical models of

the strong interaction admit the construction of a meson-diquark auxiliary-field effective action and thereby a description of baryons as loosely-bound quark-diquark composites. This is the class of theories with a chiral symmetry preserving four-fermion interaction, which includes, e.g., the Nambu–Jona-Lasinio model [6] and the Global Colour Model [7], that have been widely used in analysing low energy strong interaction phenomena.

The picture of a baryon as loosely bound quark-diquark composite can also be reached via a direct analysis of the bound state contributions to the three quark scattering matrix. The associated Schwinger function (Euclidean Green function) is just that quantity whose large Euclidean-time behaviour yields a baryon’s mass in numerical simulations of lattice-QCD. Considering the colour structure of this Schwinger function, we focus on the Clebsch-Gordon series for quarks in the fundamental representation of  $SU_c(3)$ :

$$3_c \otimes 3_c \otimes 3_c = (\bar{3}_c \oplus 6_c) \otimes 3_c = 1_c \oplus 8'_c \oplus 8_c \oplus 10_c, \quad (1)$$

from which it is clear that a *colour singlet* 3-quark contribution is only possible when two of the quarks are combined to transform according to the *antitriplet*,  $\bar{3}_c$ , representation. This is the representation under which antiquarks transform.

Single gluon exchange is repulsive in the  $6_c$  channel but attractive in the  $\bar{3}_c$  channel. It is this feature that underpins the existence of the meson-diquark bosonisation referred to above. One way to see that is to realise that the auxiliary field effective action obtained for any element of the class of four-fermion interaction models provides a Lagrangian realisation of the rainbow-ladder truncation of the Dyson-Schwinger equations (DSEs) [8]. The rainbow-ladder truncation has been widely and successfully employed in the study of meson spectroscopy and interactions, see, e.g., Refs. [9–12], and nonpointlike colour-antitriplet diquark bound states exist in this truncation of the quark-quark Bethe-Salpeter equation (BSE) [13]. Hence they provide a real degree of freedom to be used in the bosonisation.

At first sight the existence of colour-antitriplet diquark bound states in these models, and in the rainbow-ladder truncation, appears to be a problem because such states are not observed in the QCD spectrum. However, as demonstrated in Refs. [14,15], this apparent lack of confinement is primarily an artefact of the rainbow-ladder truncation. Higher order terms in the quark-quark scattering kernel, the crossed-box and vertex corrections, whose analogue in the quark-antiquark channel do not much affect many of the colour singlet meson channels, act to ensure that the quark-quark scattering matrix does not exhibit the singularities that correspond to asymptotic (unconfined) diquark bound states.

Nevertheless, such studies with improved kernels, which do not produce diquark bound states, do support a physical interpretation of the “spurious” rainbow-ladder diquark masses. Denoting the mass in a given diquark channel (scalar, pseudovector, etc.) by  $m_{qq}$ , then  $\ell_{qq} := 1/m_{qq}$  represents the range over which a true diquark correlation in this channel can persist *inside* a baryon. In this sense they are “pseudo-particle” masses that can be used to estimate which  $\bar{3}_c$  diquark correlations should dominate the bound state contribution to the

three quark scattering matrix, and hence which should be retained in deriving and solving a Poincaré covariant homogeneous Fadde'ev equation for baryons.

The simple Goldstone-theorem-preserving rainbow-ladder kernel of Ref. [16] can be used to illustrate this point. The model yields the following calculated diquark masses (isospin symmetry is assumed):

$$\frac{(qq)_{JP}}{m_{qq} \text{ (GeV)}} \left| \begin{array}{cccccccc} (ud)_{0+} & (us)_{0+} & (uu)_{1+} & (us)_{1+} & (ss)_{1+} & (uu)_{1-} & (us)_{1-} & (ss)_{1-} \\ 0.74 & 0.88 & 0.95 & 1.05 & 1.13 & 1.47 & 1.53 & 1.64 \end{array} \right. \quad (2)$$

and the results are relevant because the mass ordering is characteristic and model-independent, and lattice estimates, where available [17], agree with the masses tabulated here. Equation (2) suggests that an accurate study of the nucleon should retain the scalar and pseudovector correlations:  $(ud)_{0+}$ ,  $(uu)_{1+}$ ,  $(ud)_{1+}$ ,  $(dd)_{1+}$ , because for these diquarks  $m_{qq} \lesssim m_N$ , where  $m_N$  is the nucleon mass, but may neglect other correlations. Furthermore, it is obvious from the angular momentum Clebsch-Gordon series:  $\frac{1}{2} \otimes 0 = \frac{1}{2}$  and  $\frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2}$ , that decuplet baryons are inaccessible without pseudovector diquark correlations. It is interesting to note that  $m_{(ud)_{0+}}/m_{(uu)_{1+}} = 0.78$  cf.  $0.76 = m_N/m_\Delta$  and hence one might anticipate that the presence of diquark correlations in baryons can provide a straightforward explanation of the  $N$ - $\Delta$  mass-splitting and other like effects. These ideas were first enunciated in Refs. [18,19] and Ref. [20] provides a convincing demonstration of their efficacy.

Explicit calculations; e.g., Ref. [12], show that retaining only a scalar diquark correlation in the kernel of the nucleon's Fadde'ev equation provides insufficient binding to obtain the experimental nucleon mass: the best calculated value is typically  $\sim 40\%$  too large. However, with the addition of a pseudovector diquark it is easy to simultaneously obtain [12,21] the experimental masses of the nucleon and  $\Delta$ . Such calculations plainly verify the intuition that follows from simple mass-counting: the pseudovector diquarks are an important but subdominant element of the nucleon's Fadde'ev amplitude (cf. the scalar diquark) whilst being the sole component of the  $\Delta$ .

The presence of diquark correlations in baryons also affects the predictions for scattering observables, which may therefore provide a means for experimentally verifying the ideas described above. For example, their presence provides a simple explanation of the neutron's nonzero electric form factor [12]: charge separation arising from a heavy  $(ud)$  diquark with electric charge  $\frac{1}{3}$  holding on to a relatively light, electric charge  $(-\frac{1}{3})$   $d$ -quark. And also a prediction for the ratio of the proton's valence-quark distributions:  $d/u := d_v(x \rightarrow 1)/u_v(x \rightarrow 1)$ , which can be measured in deep inelastic scattering [22]. In this case, diquark correlations with differing masses in the nucleon's Fadde'ev amplitude are an immediate indication of the breaking of  $SU(6)$  symmetry, hence  $d/u \neq 1/2$ . Furthermore, if it were true that  $m_{(qq)_{JP}} \gg m_{(ud)_{0+}}$ , for all  $J^P \neq 0^+$ , then  $d/u = 0$ . However, as we have seen, in reality the  $1^+$  diquark is an important subdominant piece of the nucleon's Fadde'ev amplitude so that a realistic picture of diquarks in the nucleon implies  $0 < d/u < \frac{1}{2}$ , with the actual value being a sensitive measure of the proton's pseudovector diquark fraction.

## 2 Superfluidity in Quark Matter

We have outlined above the role and nature of diquark correlations in hadronic physics at zero temperature and density, and emphasised that diquarks are an idea as old as that of quarks themselves. Another phenomenon suggested immediately by the meson-diquark auxiliary-field effective action is that of diquark condensation; i.e., quark-quark Cooper pairing, which was first explored in this context using a simple version of the Nambu–Jona-Lasinio model [23]. A chemical potential promotes Cooper pairing in fermion systems and the possibility that such diquark pairing is exhibited in quark matter is also an old idea, early explorations of which employed [24] the rainbow-ladder truncation of the quark DSE (QCD gap equation). That interest in this possibility has been renewed is evident in a number of contributions to this volume [25–27]. A quark-quark Cooper pair is a composite boson with both electric and colour charge, and hence superfluidity in quark matter entails superconductivity and colour superconductivity. However, the last feature makes it difficult to identify an order parameter that can characterise a transition to the superfluid phase: the Cooper pair is gauge dependent and an order parameter is ideally describable by a gauge-invariant operator. This particularly inhibits an analysis of the phenomenon using lattice-QCD.

### 2.1 Gap Equation

Studies of the gap equation that suppress the possibility of diquark condensation show that cold, sparse two-flavour QCD exhibits a nonzero quark-antiquark condensate:  $\langle \bar{q}q \rangle \neq 0$ . If it were otherwise then the  $\pi$ -meson would be almost as massive as the  $\rho$ -meson, which would yield a very different observable world. The quark condensate is undermined by increasing  $\mu$  and  $T$ , and there is a large domain in the physical (upper-right) quadrant of the  $(\mu, T)$ -plane for which  $\langle \bar{q}q \rangle = 0$ : for the purpose of exemplification, that domain can crudely be characterised as the set (see, e.g., Refs. [28–30]):

$$\{(\mu, T) : \mu^2/\mu_c^2 + T^2/T_c^2 > 1, \mu, T > 0; \mu_c \sim 0.3\text{--}0.4\text{ GeV}, T_c \sim 0.15\text{ GeV}\}. \quad (3)$$

Increasing temperature also opposes Cooper pairing. However, since increasing  $\mu$  promotes it, there may be a (large- $\mu$ , low- $T$ )-subdomain in which quark matter exists in a superfluid phase. That domain, if it exists, is unlikely to be accessible at the Relativistic Heavy Ion Collider, because it operates in the high temperature regime, but may be realised in the core of compact astrophysical objects, which could undergo a transition to superfluid quark matter as they cool. Possible signals accompanying such a transition are considered in Refs. [25, 27].

It was observed in Ref. [31] that a direct means of determining whether a  $SU_c(N)$  gauge theory supports scalar diquark condensation is to study the gap equation satisfied by

$$\mathcal{D}(p, \mu) := \mathcal{S}(p, \mu)^{-1} = \begin{pmatrix} D(p, \mu) & \Delta^i(p, \mu) \gamma_5 \lambda_\lambda^i \\ -\Delta^i(p, -\mu) \gamma_5 \lambda_\lambda^i & CD(-p, \mu) {}^t C^\dagger \end{pmatrix}. \quad (4)$$

Here  $T = 0$ , for illustrative simplicity and because temperature can only act to destabilise a condensate, and, with  $\omega_{[\mu]} = p_4 + i\mu$ ,

$$D(p, \mu) = i\gamma \cdot \mathbf{p} A(\mathbf{p}^2, \omega_{[\mu]}^2) + B(\mathbf{p}^2, \omega_{[\mu]}^2) + i\gamma_4 \omega_{[\mu]} C(\mathbf{p}^2, \omega_{[\mu]}^2); \quad (5)$$

i.e., the inverse of the dressed-quark propagator in the absence of diquark pairing. (NB. For  $\mu = 0$ ,  $A$ ,  $B$  and  $C$  are real functions.) It is one of the fundamental features of DSE studies that the existence of a nonzero quark condensate:  $\langle \bar{q}q \rangle \neq 0$ , is signalled in the solution of the gap equation by  $B(\mathbf{p}^2, \omega_{[\mu]}^2) \neq 0$  [9].

In Eq. (4),  $\{\lambda_\lambda^i, i = 1 \dots n_c^\wedge, n_c^\wedge = N_c(N_c - 1)/2\}$  are the antisymmetric generators of  $SU_c(N_c)$  and  $C = \gamma_2\gamma_4$  is the charge conjugation matrix,

$$C\gamma_\mu^t C^\dagger = -\gamma_\mu; [C, \gamma_5] = 0, \quad (6)$$

where  $X^t$  denotes the matrix transpose of  $X$ . The key new feature here is that diquark condensation is characterised by  $\Delta^i(p, \mu) \neq 0$ , for at least one  $i$ . That is clear if one considers the quark piece of the QCD Lagrangian density:  $L[\bar{q}, q]$ . It is a scalar and hence  $L[\bar{q}, q]^t = L[\bar{q}, q]$ . Therefore  $L[\bar{q}, q] \propto L[\bar{q}, q] + L[\bar{q}, q]^t$ , and it is a simple exercise to show that this sum, and hence the action, can be re-expressed in terms of a  $2 \times 2$  diagonal matrix using the bispinor fields

$$Q(x) := \begin{pmatrix} q(x) \\ \underline{q}(x) := \tau_f^2 C \bar{q}^t \end{pmatrix}, \quad \bar{Q}(x) := (\bar{q}(x) \quad \underline{\bar{q}}(x) := q^t C \tau_f^2), \quad (7)$$

where  $\{\tau_f^i : i = 1, 2, 3\}$  are Pauli matrices that act on the isospin index.<sup>1</sup> It is plain upon inspection that a nonzero entry:  $d(x) \gamma_5$ , in row-2-column-1 of this action-matrix would act as a source for  $q^t \tau_f^2 C \gamma_5 q$ ; i.e., as a scalar diquark source.

It is plain now that the explicit  $2 \times 2$  matrix structure of  $\mathcal{D}(p, \mu)$  in Eq. (4) exhibits a quark bispinor index that is made with reference to  $Q(x)$ ,  $\bar{Q}(x)$ . This approach; i.e., employing a “matrix propagator” with “anomalous” off-diagonal elements, simply exploits the Gorkov-Nambu treatment of superconductivity in fermionic systems, which is explained in textbooks, e.g., Ref. [32]. It makes possible a well-ordered treatment and makes unnecessary a truncated bosonisation, which in all but the simplest models is a procedure difficult to improve systematically.

The bispinor gap equation can be written in the form

$$\mathcal{D}(p, \mu) = \mathcal{D}_0(p, \mu) + \begin{pmatrix} \Sigma_{11}(p, \mu) & \Sigma_{12}(p, \mu) \\ \gamma_4 \Sigma_{12}(-p, \mu) \gamma_4 & C \Sigma_{11}(-p, \mu)^t C^\dagger \end{pmatrix}, \quad (8)$$

where the second term on the right-hand-side is just the bispinor self energy. Here, in the absence of a scalar diquark source term,

$$\mathcal{D}_0(p, \mu) = (i\gamma \cdot p + m)\tau_Q^0 - \mu \tau_Q^3, \quad (9)$$

with  $m$  the current-quark mass, and the additional Pauli matrices:  $\{\tau_Q^\alpha, \alpha = 0, 1, 2, 3\}$ , act on the bispinor indices. As we will see, the structure of  $\Sigma_{ij}(p, \mu)$  specifies the theory and, in practice, also the approximation or truncation of it.

<sup>1</sup> We only consider theories with two light-flavours. Additional possibilities open if this restriction is lifted [25,26].

### 3 Two Colours

Two colour QCD (QC<sub>2</sub>D) provides an important and instructive example. In this case  $\Delta^i \lambda_\lambda^i = \Delta \tau_c^2$  in Eq. (4), with  $\frac{1}{2} \tau_c$  the generators of  $SU_c(2)$ , and it is useful to employ a modified bispinor

$$Q_2(x) := \begin{pmatrix} q(x) \\ \underline{q}_2 := \tau_c^2 \underline{q}(x) \end{pmatrix}, \quad \bar{Q}_2(x) := (\bar{q}(x) \quad \underline{\bar{q}}_2(x) := \underline{\bar{q}}(x) \tau_c^2). \quad (10)$$

Embedding the additional factor of  $\tau_c^2$  in this way makes it possible to write the Lagrangian's fermion–gauge–boson interaction term as

$$\bar{Q}_2(x) \frac{i}{2} g \gamma_\mu \tau_c^k \tau_Q^0 Q_2(x) A_\mu^k(x) \quad (11)$$

because  $SU_c(2)$  is pseudoreal; i.e.,  $\tau_c^2 (-\tau_c)^t \tau_c^2 = \tau_c$ , and the fundamental and conjugate representations are equivalent; i.e., fermions and antifermions are practically indistinguishable. (That the interaction term takes this form is easily seen using  $L[\bar{q}, q]^t = L[\bar{q}, q]$ .)

Using the pseudoreality of  $SU_c(2)$  it can be shown that, for  $\mu = 0$  and in the chiral limit,  $m = 0$ , the general solution of the bispinor gap equation is [31]

$$\mathcal{D}(p) = i\gamma \cdot p A(p^2) + \mathcal{V}(-\boldsymbol{\pi}) \mathcal{M}(p^2), \quad \mathcal{V}(\boldsymbol{\pi}) = \exp \left\{ i\gamma_5 \sum_{\ell=1}^5 T^\ell \pi^\ell \right\} = \mathcal{V}(-\boldsymbol{\pi})^{-1}, \quad (12)$$

where  $\pi^{\ell=1,\dots,5}$  are arbitrary constants and

$$\{T^{1,2,3} = \tau_Q^3 \otimes \tau_f, T^4 = \tau_Q^1 \otimes \tau_f^0, T^5 = \tau_Q^2 \otimes \tau_f^0\}, \quad \{T^i, T^j\} = 2\delta^{ij}, \quad (13)$$

so that  $\mathcal{D}^{-1}$  is

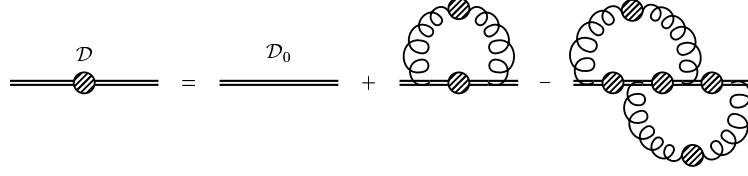
$$\mathcal{S}(p) = \frac{-i\gamma \cdot p A(p^2) + \mathcal{V}(\boldsymbol{\pi}) \mathcal{M}(p^2)}{p^2 A^2(p^2) + \mathcal{M}^2(p^2)} := -i\gamma \cdot p \sigma_V(p^2) + \mathcal{V}(\boldsymbol{\pi}) \sigma_S(p^2). \quad (14)$$

[To illustrate this, note that inserting  $\boldsymbol{\pi} = (0, 0, 0, 0, -\frac{1}{4}\pi)$  produces an inverse bispinor propagator with the simple form in Eq. (4).]

That the gap equation is satisfied for any constants  $\pi^\ell$  signals a vacuum degeneracy – it corresponds to a multidimensional “Mexican hat” structure of the theory’s effective potential, as noted in a related context in Ref. [33]. Consequently, if the interaction supports a mass gap, then that gap describes a five-parameter continuum of degenerate condensates:

$$\langle \bar{Q}_2 \mathcal{V}(\boldsymbol{\pi}) Q_2 \rangle \neq 0, \quad (15)$$

and there are 5 associated Goldstone bosons: 3 pions, a diquark and an anti-diquark. (Diquarks are the “baryons” of QC<sub>2</sub>D.) In the construction of Eq. (12) one has a simple elucidation of a necessary consequence of the Pauli–Gürsey symmetry of QC<sub>2</sub>D; i.e., the practical equivalence of particles and antiparticles.



**Fig. 1.** Illustration of the vertex-corrected gap equation, which is the next-to-leading-order in the systematic, symmetry-preserving truncation scheme of Ref. [14]. Retaining only the first two diagrams on the right-hand-side yields the dressed-rainbow truncation. Each bispinor quark-gluon vertex is bare but the shaded circles mark quark and gluon 2-point functions that are dressed. The corresponding truncation in the relevant Bethe-Salpeter equations ensures the absence of diquark bound states in the strong interaction spectrum. (Adapted from Ref. [31].)

For  $m \neq 0$ , the gap equation requires [31]  $\text{tr}_{FQ} [T^i \mathcal{V}] = 0$ , so that in this case only  $\langle \bar{Q}_2 Q_2 \rangle \neq 0$  and now the spectrum contains five degenerate but massive pseudo-Goldstone bosons. This illustrates that a nonzero current-quark mass promotes a quark condensate and opposes diquark condensation.

For  $\mu \neq 0$  the general solution of the gap equation has the form

$$\mathcal{D}(p, \mu) = \begin{pmatrix} D(p, \mu) & \gamma_5 \Delta(p, \mu) \\ -\gamma_5 \Delta^*(p, \mu) & CD(-p, \mu)C^\dagger \end{pmatrix}. \quad (16)$$

In the *absence* of a diquark condensate; i.e., for  $\Delta \equiv 0$ ,

$$[U_B(\alpha), \mathcal{D}(p, \mu)] = 0, \quad U_B(\alpha) := e^{i\alpha\tau_Q^3 \otimes \tau_f^0}; \quad (17)$$

i.e., baryon number is conserved in QC<sub>2</sub>D. This makes plain that the existence of a diquark condensate dynamically breaks this symmetry.

To proceed we choose to be explicit and employ the dressed-rainbow truncation of the gap equation, see Fig. 1, with a model for the Landau gauge dressed-gluon propagator:

$$g^2 D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \mathcal{F}_2(k^2), \quad \mathcal{F}_2(k^2) = \frac{64}{9} \pi^4 \hat{\eta}^2 \delta^4(k). \quad (18)$$

This form for  $g^2 D_{\mu\nu}(k)$  was introduced [34] for the modelling of confinement in QCD but it is also appropriate here because the string tension in QC<sub>2</sub>D is nonzero, and that is represented implicitly in Eq. (18) via the mass-scale  $\hat{\eta}$ .

Using Eq. (18) we obtain an algebraic dressed-rainbow gap equation that, for  $p^2 = |\mathbf{p}|^2 + p_4^2 = 0$ , reads:

$$A - 1 = \frac{1}{2} \hat{\eta}^2 K \{ A (B^{*2} - C^{*2} \mu^2) + A^* |\Delta|^2 \}, \quad (19)$$

$$(C - 1) \mu = \frac{\mu}{2} \hat{\eta}^2 K \{ C (B^{*2} - C^{*2} \mu^2) - C^* |\Delta|^2 \}, \quad (20)$$

$$B - m = \hat{\eta}^2 K \{ B (B^{*2} - C^{*2} \mu^2) + B^* |\Delta|^2 \}, \quad (21)$$

$$\Delta = \hat{\eta}^2 K \{ \Delta (|B|^2 + |C|^2 \mu^2) + \Delta |\Delta|^2 \}, \quad (22)$$

with  $K^{-1} = |B^2 - C^2\mu^2|^2 + 2|\Delta|^2(|B|^2 + |C|^2\mu^2) + |\Delta|^4$ . These equations possess a  $B \leftrightarrow \Delta$  symmetry when  $(m, \mu) = 0$ , which is a straightforward illustration of the vacuum degeneracy described above using the matrix  $\mathcal{V}(\boldsymbol{\pi})$ . (Recall that for  $\mu = 0$ ,  $A$ ,  $B$  and  $C$  are real functions.) They also exemplify the general result that  $\Delta$  is real for all  $\mu$ . Another exemplary result follows from a linearisation in  $\mu^2$ :  $\mu \neq 0$  acts to promote a nonzero value of  $\Delta$  but oppose a nonzero value of  $B$ ; i.e., a nonzero chemical potential plainly acts to promote Cooper pairing at the expense of  $\langle \bar{q}q \rangle$ .

For  $(m, \mu) = 0$  the solution of the dressed-rainbow gap equation obtained using Eq. (18) is:

$$A(p^2) = C(p^2) = \begin{cases} 2, & p^2 < \frac{\hat{\eta}^2}{4} \\ \frac{1}{2} \left( 1 + \sqrt{1 + \frac{2\hat{\eta}^2}{p^2}} \right), & \text{otherwise,} \end{cases} \quad (23)$$

$$\mathcal{M}^2(p^2) := B^2(p^2) + \Delta^2(p^2) = \begin{cases} \hat{\eta}^2 - 4p^2, & p^2 < \frac{\hat{\eta}^2}{4} \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

As we have already mentioned, the dynamically generated mass function,  $\mathcal{M}(p^2)$ , is tied to the existence of quark and/or diquark condensates, which can be illustrated by noting that  $(B = 0, \Delta \neq 0)$  corresponds to  $\boldsymbol{\pi} = (0, 0, 0, 0, \frac{1}{2}\pi)$  in Eq. (15); i.e.,  $\langle \bar{Q}_2 i\gamma_5 \tau_Q^2 Q_2 \rangle \neq 0$ , while  $(B \neq 0, \Delta = 0)$  corresponds to  $\boldsymbol{\pi} = (0, 0, 0, 0, 0)$ ; i.e.,  $\langle \bar{Q}_2 Q_2 \rangle \neq 0$ .

The usual chiral,  $SU_A(2)$ , transformations are realised via

$$\mathcal{D}(p, \mu) \rightarrow V(\boldsymbol{\pi}) \mathcal{D}(p, \mu) V(\boldsymbol{\pi}), \quad V(\boldsymbol{\pi}) := e^{i\gamma_5 \cdot \boldsymbol{\pi}}, \quad \boldsymbol{\pi} = (\pi^1, \pi^2, \pi^3), \quad (25)$$

and therefore, since the anticommutator  $\{\boldsymbol{T}, T^{4,5}\} = 0$ , a diquark condensate does not dynamically break chiral symmetry. On the other hand, since  $[\mathbf{1}, \boldsymbol{T}] = 0$ , a quark condensate does dynamically break chiral symmetry.

In addition, and of particular importance, is the feature that in *combination* with the momentum-dependent vector self energy the momentum-dependence of  $\mathcal{M}(p^2)$  ensures that the dressed-quark propagator does not have a Lehmann representation and hence can be interpreted as describing a confined quark [8–10]. The interplay between the scalar and vector self energies is the key to this realisation of confinement. The qualitative features of this simple model's dressed-quark propagator have been confirmed in recent lattice-QCD simulations [35] and the agreement between those simulations and more sophisticated DSE studies is semi-quantitative [11].

In the steepest descent (or stationary phase) approximation the contribution of dressed-quarks to the thermodynamic pressure is

$$p_\Sigma(\mu, T) = \frac{1}{2\beta V} \left\{ \text{Tr} \text{Ln} [\beta \mathcal{S}^{-1}] - \frac{1}{2} \text{Tr} [\Sigma \mathcal{S}] \right\}, \quad (26)$$

where  $\beta = 1/T$ , and “Tr” and “Ln” are extensions of “tr” and “ln” to matrix-valued functions.



The MIT Bag Model pictures the quarks in a baryon as occupying a spatial volume from which the nontrivial quark-condensed vacuum (scalar-field) has been expelled. Therefore, as observed in Refs. [33], the bag constant can be identified as the pressure difference between the  $\langle \bar{q}q \rangle \neq 0$  vacuum, the so-called Nambu-Goldstone phase in which chiral symmetry is dynamically broken, and the chirally symmetric no-condensate alternative, which is called the Wigner-Weyl vacuum. That difference is given by

$$\mathcal{B}_B(\mu) := p_\Sigma(\mu, \mathcal{S}[B, \Delta = 0]) - p_\Sigma(\mu, \mathcal{S}[B = 0, \Delta = 0]), \quad (27)$$

and it is, of course,  $\mu$ -dependent because the vacuum *evolves* with changing  $\mu$ .  $\mathcal{B}_B$  also evolves with temperature and this necessary  $(\mu, T)$ -dependence of the bag constant can have an important effect on quark star properties; e.g., reducing the maximum supportable mass of a quark matter star, as discussed in Ref. [36].

If we define, by analogy,

$$\mathcal{B}_\Delta(\mu) := p_\Sigma(\mu, \mathcal{S}[B = 0, \Delta]) - p_\Sigma(\mu, \mathcal{S}[B = 0, \Delta = 0]), \quad (28)$$

then the relative stability of the quark- and diquark-condensed phases is measured by the pressure difference

$$\delta p(\mu) := \mathcal{B}_\Delta(\mu) - \mathcal{B}_B(\mu). \quad (29)$$

For  $\delta p(\mu) > 0$  the diquark condensed phase is favoured.

At  $(m = 0, \mu = 0)$ ,  $\delta p = 0$ , with

$$\mathcal{B}_B(0) = \mathcal{B}_\Delta(0) = (0.092 \hat{\eta})^4. \quad (30)$$

This equality is a manifestation of the vacuum degeneracy identified above in connection with the matrix  $\mathcal{V}(\boldsymbol{\pi})$ . However,

$$\text{with } m = 0, \delta p > 0 \text{ for all } \mu > 0, \quad (31)$$

which means that the Wigner-Weyl vacuum is unstable with respect to diquark condensation for all  $\mu > 0$  [31] and that the superfluid phase is favoured over the Nambu-Goldstone phase.

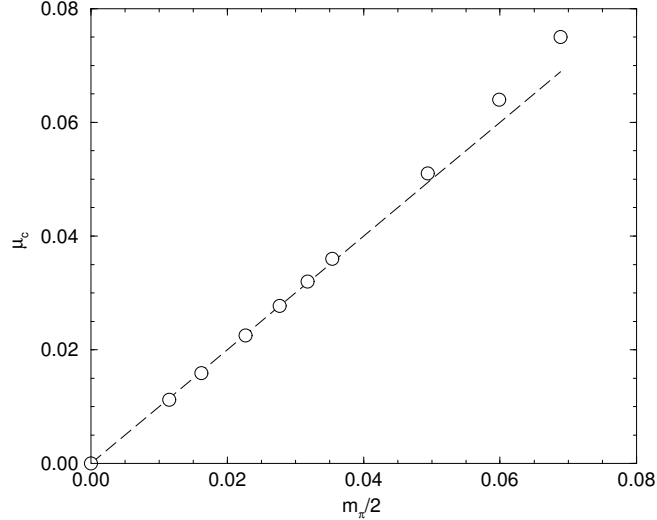
Now, although the action for the  $\mu \neq 0$  theory is invariant under

$$Q_2 \rightarrow U_B(\alpha) Q_2, \quad \bar{Q}_2 \rightarrow \bar{Q}_2 U_B(-\alpha), \quad (32)$$

which is associated with baryon number conservation, the diquark condensate breaks this symmetry:

$$\langle \bar{Q}_2 i \gamma_5 \tau_Q^2 Q_2 \rangle \rightarrow \cos(2\alpha) \langle \bar{Q}_2 i \gamma_5 \tau_Q^2 Q_2 \rangle - \sin(2\alpha) \langle \bar{Q}_2 i \gamma_5 \tau_Q^1 Q_2 \rangle; \quad (33)$$

i.e., it is a ground state that is not invariant under the transformation. Hence, for  $(m = 0, \mu \neq 0)$ , only one Goldstone mode remains. These symmetry breaking patterns and the concomitant numbers of Goldstone modes in QC<sub>2</sub>D are also described in Ref. [37].



**Fig. 2.** Evolution of the critical chemical potential for diquark condensation as the current-quark mass is increased. The coordinate measures the magnitude of the current-quark mass through the mass of the theory’s lightest excitation [a pseudo-Goldstone mode, as described after Eq. (15)].  $\mu_c$  and  $m_\pi$  are measured in units of the model’s mass scale,  $\hat{\eta}$  in Eq. (18): for  $m = 0$ , the vector meson mass is  $\frac{1}{\sqrt{2}}\hat{\eta}$ .

For  $m \neq 0$  and small values of  $\mu$  the gap equation only admits a solution with  $\Delta \equiv 0$ ; i.e., diquark condensation is blocked because the current-quark mass is a source of the quark condensate [see, e.g., Eq. (21) and the comments after Eq. (15)]. However, with increasing  $\mu$ , the theory undergoes a transition to a phase in which the diquark condensate is nonzero. We identify the transition as second order because the diquark condensate falls continuously to zero as  $\mu \rightarrow \mu_c^+$ , where  $\mu_c$  is the critical chemical potential. In Fig. 2 we plot the critical chemical potential as a function of  $m_\pi/2$ , where, to sidestep solving the Bethe-Salpeter equation,  $m_\pi$  was obtained using a Gell-Mann–Oakes–Renner-like mass formula, Eqs. (16)–(18) in Ref. [29], which follows [38] from the axial-vector Ward-Takahashi identity. From the figure it is clear that this simple model of QC<sub>2</sub>D exhibits the relation

$$\mu_c = \frac{1}{2} m_\pi, \quad (34)$$

which is anticipated for QCD-like theories with pseudoreal fermions [39]. We note that the deviation from Eq. (34) at larger values of  $m_\pi$  results from neglecting  $O(m^2)$ -corrections in the mass formula. This omission leads to an underestimate of the pion mass [40], which is responsible for the upward deflection of the calculated results evident in Fig. 2.

In exemplifying these features we have employed the rainbow-ladder truncation. However, improving on that will only yield quantitative changes of  $\lesssim 20\%$  in the results because the pseudoreality of QC<sub>2</sub>D and the equal dimension of the colour and bispinor spaces, which underly the theory’s Pauli–Gürsey symmetry,

ensure that the entire discussion remains qualitatively unchanged. In particular, the results of Fig. 2 and Eq. (34), being tied to chiral symmetry, remain unchanged because at least one systematic, chiral symmetry preserving truncation scheme exists [14].

## 4 Three Colours

The exploration of superfluidity in true QCD encounters two differences: the dimension of the colour space is greater than that of the bispinor space and the fundamental and conjugate representations of the gauge group are not equivalent. The latter is of obvious importance because it entails that the quark-quark and quark-antiquark scattering matrices are qualitatively different.

$n_c^\wedge = 3$  in QCD and hence in canvassing superfluidity it is necessary to choose a direction for the condensate in colour space;<sup>2</sup> e.g.,  $\Delta^i \lambda_\Lambda^i = \Delta \lambda^2$  in Eq. (4), so that

$$\mathcal{D}(p, \mu) = \left( \begin{array}{c|c} D_\parallel(p, \mu)P_\parallel + D_\perp(p, \mu)P_\perp & \Delta(p, \mu)\gamma_5\lambda^2 \\ \hline -\Delta(p, -\mu)\gamma_5\lambda^2 & CD_\parallel(-p, \mu)C^\dagger P_\parallel + CD_\perp(p, \mu)C^\dagger P_\perp \end{array} \right), \quad (35)$$

where  $P_\parallel = (\lambda^2)^2$ ,  $P_\perp + P_\parallel = \text{diag}(1, 1, 1)$ , and  $D_\parallel$ ,  $D_\perp$  are defined via obvious generalisations of Eqs. (4), (5). In Eq. (35) the evident, demarcated block structure makes explicit the bispinor index: each block is a  $3 \times 3$  colour matrix and the subscripts:  $\parallel$ ,  $\perp$ , indicate whether or not the subspace is accessible via  $\lambda_2$ .

The bispinors associated with this representation are given in Eqs. (7) and in this case the Lagrangian's quark-gluon interaction term is

$$\bar{Q}(x)ig\Gamma_\mu^a Q(x)A_\mu^a(x), \quad \Gamma_\mu^a = \left( \begin{array}{c|c} \frac{1}{2}\gamma_\mu\lambda^a & 0 \\ \hline 0 & -\frac{1}{2}\gamma_\mu(\lambda^a)^t \end{array} \right). \quad (36)$$

It is instructive to compare this with Eq. (11): with three colours the interaction term is not proportional to the identity matrix in the bispinor space,  $\tau_Q^0$ . This makes plain the inequivalence of the fundamental and conjugate fermion representations of  $SU_c(3)$ , which entails that quark-antiquark scattering is different from quark-quark scattering.

It is straightforward to derive the gap equation at arbitrary order in the truncation scheme of Ref. [14] and it is important to note that because

$$D_\parallel(p, \mu)P_\parallel + D_\perp(p, \mu)P_\perp = \lambda^0 \left\{ \frac{2}{3}D_\parallel(p, \mu) + \frac{1}{3}D_\perp(p, \mu) \right\} + \frac{1}{\sqrt{3}}\lambda^8 \left\{ D_\parallel(p, \mu) - D_\perp(p, \mu) \right\} \quad (37)$$

the interaction:  $\Gamma_\mu^a \mathcal{S}(p, \mu) \Gamma_\nu^a$ , necessarily couples the  $\parallel$ - and  $\perp$ -components. Reference [31] explored the possibility of diquark condensation in QCD using both

<sup>2</sup> It is this selection of a direction in colour space that opens the possibility for colour-flavour locked diquark condensation in a theory with three effectively-massless quarks; i.e., current-quark masses  $\ll \mu$  [25,26].

the rainbow and vertex-corrected gap equation, illustrated in Fig. 1, with

$$g^2 D_{\mu\nu}(k) = \left( \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \mathcal{F}(k^2), \quad \mathcal{F}(k^2) = 4\pi^4 \eta^2 \delta^4(k). \quad (38)$$

For  $(m, \mu) = 0$  the rainbow-ladder truncation yields

$$m_\omega^2 = m_\rho^2 = \frac{1}{2} \eta^2, \quad \langle \bar{q}q \rangle^0 = (0.11 \eta)^3, \quad \mathcal{B}_B(\mu = 0) = (0.10 \eta)^4, \quad (39)$$

and momentum-dependent vector self energies, which lead to an interaction between the  $\parallel$ - and  $\perp$ -components of  $\mathcal{D}$  that blocks diquark condensation. This is in spite of the fact that  $\lambda^a \lambda^2 (-\lambda^a)^t = \frac{1}{2} \lambda^a \lambda^a$ , which entails that the rainbow-truncation quark-quark scattering kernel is purely attractive and strong enough to produce diquark bound states [13]. (Remember that in the colour singlet meson channel the rainbow-ladder truncation gives the colour coupling  $\lambda^a \lambda^a$ ; i.e., an interaction with the same sign but twice as strong.)

For  $\mu \neq 0$  and in the *absence* of diquark condensation this model and truncation exhibits [28] coincident, first order chiral symmetry restoring and deconfining transitions at

$$\mu_{c, \text{rainbow}}^{B, \Delta=0} = 0.28 \eta = 0.3 \text{ GeV}, \quad (40)$$

with  $\eta = 1.06 \text{ GeV}$  fixed by fitting the  $m \neq 0$  vector meson mass [14].

For  $(m = 0, \mu \neq 0)$ , however, the rainbow-truncation gap equation admits a solution with  $\Delta(p, \mu) \neq 0$  and  $B(p, \mu) \equiv 0$ . The pressure difference,  $\delta p(\mu)$  in Eq. (29), is again the way to determine whether the stable ground state is the Nambu-Goldstone or superfluid phase. With increasing  $\mu$ ,  $\mathcal{B}_B(\mu)$  decreases, very slowly at first, and  $\mathcal{B}_\Delta(\mu)$  increases rapidly from zero. That evolution continues until

$$\mu_{c, \text{rainbow}}^{B=0, \Delta} = 0.25 \eta = 0.89 \mu_{c, \text{rainbow}}^{B, \Delta=0}, \quad (41)$$

where  $\mathcal{B}_\Delta(\mu)$  becomes greater-than  $\mathcal{B}_B(\mu)$ . This signals a first order transition to the superfluid ground state and at the boundary

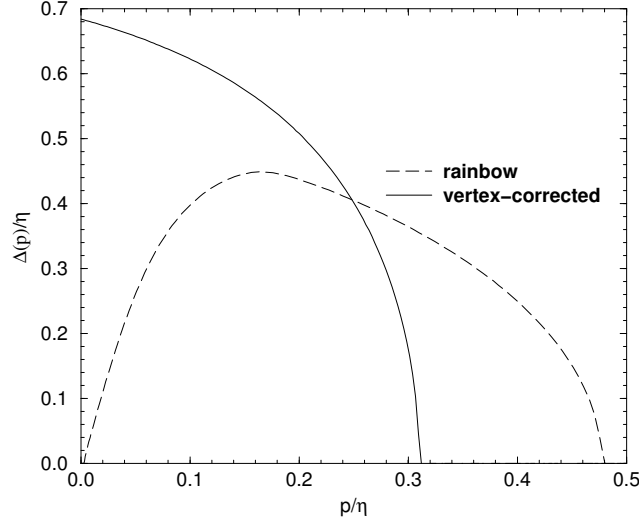
$$\langle \bar{Q} i \gamma_5 \tau_Q^2 \lambda^2 Q \rangle_{\mu=\mu_{c, \text{rainbow}}^{B=0, \Delta}} = (0.65)^3 \langle \bar{Q} Q \rangle_{\mu=0}. \quad (42)$$

Since  $\mathcal{B}_\Delta(\mu) > 0$  for all  $\mu > 0$  there is no intermediate domain of  $\mu$  in which all condensates vanish.

The solution of the rainbow gap equation:  $\Delta(p, \mu_c^{B, \Delta})$ , which is real and characterises the diquark gap, is plotted in Fig. 3. It vanishes at  $p^2 = 0$  as a consequence of the  $\parallel$ - $\perp$  coupling that blocked diquark condensation at  $\mu = 0$ , and also at large  $p^2$ , which is a manifestation of this simple model's version of asymptotic freedom.

The chemical potential<sup>3</sup> at which the switch to the superfluid ground state occurs, Eq. (41), is consistent with other estimates made using models compa-

<sup>3</sup> We note that in a two flavour free-quark gas at  $\mu = 0.3 \text{ GeV}$  the baryon number density is  $1.5 \rho_0$ , where  $\rho_0 = 0.16 \text{ fm}^{-3}$ . In the same system at  $\mu = 0.55 \text{ GeV}$  the baryon number density is  $> 10 \rho_0$ . For comparison, the central core density expected in a  $1.4 M_\odot$  neutron star is  $3.6\text{--}4.1 \rho_0$  [41]. Arguments valid at “asymptotically large” quark chemical potential are therefore unlikely to be relevant to experimentally or observationally accessible systems.



**Fig. 3.** Dashed line:  $\Delta(z, \mu_c^{B,\Delta})$  obtained in rainbow truncation with the QCD model defined via Eq. (38), plotted for  $\alpha = 0$  as a function of  $p$ , where  $z = p(0, 0, \sin \alpha, i\mu + \cos \alpha)$ . As  $\mu$  increases, the peak position shifts to larger values of  $p$  and the peak height increases. Solid line:  $\Delta(z, \mu = 0)$  obtained as the solution of Eq. (43), the vertex-corrected gap equation, also with  $\alpha = 0$ . (Adapted from Ref. [31].)

rable to the rainbow-truncation class [25,26,42–44], as is the large magnitude of the gap at this point [25,26,42,43].

A question that now arises is: How sensitive is this phenomenon to the nature of the quark-quark interaction? As we discussed in connection with Eq. (2), the inhomogeneous dressed-ladder BSE exhibits particle-like singularities in the  $0^+$  diquark channels and such states do not exist in the strong interaction spectrum. Does diquark condensation persist when a truncation of the gap equation is employed that does not correspond to a BSE whose solutions exhibit diquark bound states? The vertex corrected gap equation,

$$\mathcal{D}(p, \mu) = \mathcal{D}_0(p, \mu) + \frac{3}{16}\eta^2 \Gamma_\rho^a \mathcal{S}(p, \mu) \Gamma_\rho^a - \frac{9}{256}\eta^4 \Gamma_\rho^a \mathcal{S}(p, \mu) \Gamma_\sigma^b \mathcal{S}(p, \mu) \Gamma_\rho^a \mathcal{S}(p, \mu) \Gamma_\sigma^b, \quad (43)$$

which is depicted in Fig. 1, is just such a truncation and it was also studied in Ref. [31].

In this case there is a  $\Delta \neq 0$  solution even for  $\mu = 0$ , which is illustrated in Fig. 3, and using the interaction of Eq. (38)

$$m_\rho^2 = (1.1) m_\rho^2 \text{ ladder}, \quad \langle \bar{Q}Q \rangle = (1.0)^3 \langle \bar{Q}Q \rangle^{\text{rainbow}}, \quad \mathcal{B}_B = (1.1)^4 \mathcal{B}_B^{\text{rainbow}}, \quad (44)$$

where the rainbow-ladder results are given in Eqs. (39), and

$$\langle \bar{Q} i \gamma_5 \tau_Q^2 \lambda^2 Q \rangle = (0.48)^3 \langle \bar{Q}Q \rangle, \quad \mathcal{B}_\Delta = (0.42)^4 \mathcal{B}_B. \quad (45)$$

The last result shows, unsurprisingly, that the Nambu-Goldstone phase is favoured at  $\mu = 0$ . *Precluding* diquark condensation, the solution of the vertex-corrected gap equation exhibits coincident, first order chiral symmetry restoring and deconfinement transitions at

$$\mu_c^{B,\Delta=0} = 0.77 \mu_{c,\text{rainbow}}^{B,\Delta=0}. \quad (46)$$

Admitting diquark condensation, however, the  $\mu$ -dependence of the bag constants again shows there is a first order transition to the superfluid phase, here at

$$\mu_c^{B=0,\Delta} = 0.63 \mu_c^{B,\Delta=0}, \text{ with } \langle \bar{Q} i \gamma_5 \tau_Q^2 \lambda^2 Q \rangle_{\mu=0.63 \mu_c^{B,\Delta=0}} = (0.51)^3 \langle \bar{Q} Q \rangle_{\mu=0}. \quad (47)$$

(NB. This discussion is still for  $m = 0$ . We saw at the end of Sec. 3 what effects to anticipate at  $m \neq 0$ .) Thus the material step of employing a truncation that eliminates diquark bound states leads only to small quantitative changes in the quantities characterising the still extant superfluid phase; e.g., reductions in the magnitude of both the critical chemical potential for the transition to superfluid quark matter and the gap. Hence scalar diquark condensation appears to be a robust phenomenon. One caveat to bear in mind, however, is that the gap equation studies conducted hitherto do not obviate the question of whether the diquark condensed phase is stable with-respect-to dinucleon condensation [45], which requires further attention.

Heating causes the diquark condensate to evaporate. Existing studies suggest that it will disappear for  $T \gtrsim 60\text{--}100$  MeV [25,26,42,44]. However, such temperatures are high relative to that anticipated inside dense astrophysical objects, which may indeed therefore provide an environment for detecting quark matter superfluidity.

## 5 Summary

The idea that diquark correlations play an important role in strong interaction physics is an old one. However, modern computational resources and theoretical techniques make possible a more thorough and quantitative exploration of the merits of this idea and its realisation in QCD. These advances are in part responsible for the contemporary resurgence of interest in all aspects of diquark-related phenomena.

Herein we have attempted to provide a qualitative understanding of the nature of diquark condensation using exemplary, algebraic models, and focusing on two flavour theories for simplicity.

The gap equation is a primary tool in all studies of pairing. Using the special case of 2-colour QCD, QC<sub>2</sub>D, we illustrated via an analysis of the gap equation how a nonzero chemical potential promotes Cooper pairing and how that pairing can overwhelm a source for quark-antiquark condensation, such as a fermion current-mass. As we saw, the pseudoreality of  $SU(N_c = 2)$  entails that QC<sub>2</sub>D

has a number of special symmetry properties, which dramatically affect the spectrum.

Turning to QCD itself, we saw that one can expect a nonzero quark condensate at zero chemical potential:  $\sigma := -\langle \bar{q}q \rangle^{1/3} \neq 0$ , to give way to a diquark condensate when the chemical potential exceeds  $\approx 2\sigma$ , and at this point the diquark gap is  $\approx \sigma/2$ . The diquark condensate melts when the temperature exceeds  $\sim 60-100$  MeV; i.e., one-third to one-half of the chiral symmetry restoring temperature in two-flavour QCD. These features are model-independent in the sense that the many, disparate models applied recently to the problem yield results in semi-quantitative agreement.

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